Topological Data Analysis
MVA

Last name:
First name:

Duration: 30 minutes.

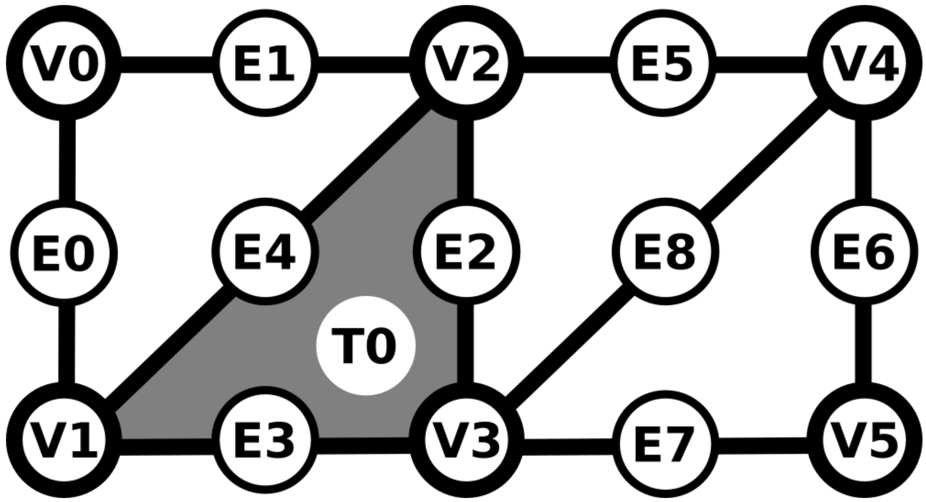

Figure 1: Simplicial complex $\mathcal{K}$, made of 6 vertices, 9 edges and 1 triangle.

Question 1: Let $\mathcal{K}$ be the simplicial complex shown in Figure 1. Give the Betti numbers of $\mathcal{K}$.
a) $\beta_0 = 1$, $\beta_1 = 3$, $\beta_2 = 1$, $\beta_i = 0, \forall i > 2$.
b) $\beta_0 = 1$, $\beta_1 = 3$, $\beta_i = 0, \forall i > 1$.
c) $\beta_0 = 1$, $\beta_1 = 5$, $\beta_i = 0, \forall i > 1$.

Question 2: Let $\mathcal{M}$ be the triangulation of a 3-manifold embedded in $\mathbb{R}^3$. For every interior $2-$simplex $\sigma_2 \in \mathcal{M}$, what is $\beta_0(Lk(\sigma_2))$ equal to?
a) $\beta_0(Lk(\sigma_2)) = 0$
b) $\beta_0(Lk(\sigma_2)) = 1$
c) $\beta_0(Lk(\sigma_2)) = 2$

Question 3: Let $f : \mathcal{M} \rightarrow \mathbb{R}$ be a piecewise linear (PL) scalar function defined on a closed, orientable PL 2-manifold $\mathcal{M}$, admitting a *monkey* saddle on the vertex v_s. What are $\beta_0(Lk^-(v_s))$ and $\beta_0(Lk^+(v_s))$ equal to?
a) $\beta_0(Lk^-(v_s)) \geq 3$ and $\beta_0(Lk^+(v_s)) \geq 3$
b) $\beta_0(Lk^-(v_s)) = 2$ and $\beta_0(Lk^+(v_s)) = 2$
c) $\beta_0(Lk^-(v_s)) = 1$ and $\beta_0(Lk^+(v_s)) = 1$

Question 4: Let $f : \mathcal{M} \rightarrow \mathbb{R}$ be a PL *Morse* function defined on a closed, orientable PL 2-manifold $\mathcal{M}$ of genus 5. What is the number of loops of the

Reeb graph $\mathcal{R}(f)$ of f?
a) 10
b) 5
c) 1

Question 5: In Discrete Morse Theory, given a discrete gradient field F defined on a PL d-manifold $\mathcal{M}$, what is the dimension of the simplices on which local maxima can occur?
a) 0
b) $d/2$
c) d

Question 6: To compute the 1-dimensional persistent homology of a 2-dimensional simplicial complex, it is enough to know its 1-dimensional skeleton.
a) True
b) False

Question 7: Let D_1 and D_2 be the two persistence diagrams defined by:

$$D_1 = [(1, (1.0, 2.0)), (1, (2.0, 4.0)), (1, (3.0, 3.5))] \tag{1}$$

and

$$D_2 = [(1, (1.8, 4.2)), (1, (2.7, 3.5))]. \tag{2}$$

The bottleneck distance between D_1 and D_2 is equal to:
a) 0.2
b) 0.3
c) 0.5
d) 1.0

Question 8: What is the dimension of the Vietoris-Rips complex of radius $r = 2$ built on top of the 4 vertices $(0, 0), (1, 0), (1, 1), (0, 1)$ of the unit square in the plane $\mathbb{R}^2$?
a) 1
b) 2
c) 3
d) 4

Question 9: The 2-dimensional unit sphere $\mathbb{S}^2$ in $\mathbb{R}^3$ is not homeomorphic to the 2-dimensional torus $\mathbb{T}^2$ (cartesian product of 2 unit circles $\mathbb{S}^1 \times \mathbb{S}^1$ because:
a) $\beta_0(\mathbb{S}^2) \neq \beta_0(\mathbb{T}^2)$
b) $\beta_1(\mathbb{S}^2) \neq \beta_1(\mathbb{T}^2)$
c) $\beta_2(\mathbb{S}^2) \neq \beta_2(\mathbb{T}^2)$
d) $\beta_3(\mathbb{S}^2) \neq \beta_3(\mathbb{T}^2)$

Question 10: Given a data set of $1,000,000$ of points in $\mathbb{R}^3$, is it realistic to compute the persistent homology of the Vietoris-Rips filtration built on top of this data set?
a) Yes
b) No. In that case, propose a possible solution to overcome the problem.